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Mild Inflation and Modified General Relativity in the Early Universe

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This article deals with particle creation and the production of specific entropy per baryon in the early universe, which is regarded as a thermodynamically open system in the sense of Prigogine. The modified general relativity (MGR) theory of Rastall, Al-Rawaf, and Taha is employed. It contains an extra independent constant η which is peculiar to the non-Newtonian regime, besides the usual gravitational constant. Usual general relativity (GR) appears here as a special case for $\eta = 1$. With a modified thermodynamic energy conservation law, it is possible to obtain an equation for the expansion scalar by incorporating the epoch dependence of elementary particle masses. The epoch dependence of particle masses for the Robertson–Walker (RW) universe appears as a consequence of hadronic matter extension in a microlocal space-time regarded as anisotropic and Finslerian. The governing equations in the present formalism specify the equation of state and give a solution for the expansion scalar. This solution represents a mild inflationary phase in the very early universe. It is also shown that there are no 'turn-on' and 'turn-off' problems for this mild inflation. It can account for particle creation and production of specific entropy per baryon consistent with the observation. The production of specific entropy per baryon is also considered here in the MGR framework with the introduction of viscous pressure; the calculated value is in good agreement with observation for the GR case, but for the MGR case, in order to have its value within observational limits, η must lie in the range $0.75 \leq \eta \leq 1$. It is also argued that this formalism does not have horizon and flatness problems.

1. INTRODUCTION

The theoretical foundations of covariant energy-momentum conservation in curved space-time were put into doubt by Rastall (1972). He modified it with the assumption that the divergence of the energy-momentum tensor might be dependent on the curvature. Actually, it was thought to be proportional to the gradient of the scalar curvature. This scalar curvature, of course, vanishes

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for flat space-time, and thus in special relativity the conservation of energy-momentum is restored. The modified general relativity (MGR) thus obtained is found to be equivalent to the gravitational field equations derived recently by Al-Rawaf and Taha (1996a, b) with the use of conventional heuristic methods (Weinberg, 1972), but not requiring energy-momentum conservation. The field equations of the MGR of Rastall, Al-Rawaf, and Taha cannot be derived from a variational principle. However, a prototype of this MGR can be derived from a variational principle, but it contains a variable gravitational 'constant' (Smalley, 1984). Interestingly, the formulation of Al-Rawaf and Taha contains two independent fundamental constants, one of which is the usual Newton constant and the other is an adjustable parameter η satisfying $0 \leq \eta \leq 1$. This constant η may be regarded as characteristic of the non-Newtonian regime, and standard general relativity (GR) is obtained for $\eta = 1$. The nonconservation of energy-momentum in MGR may be looked at in a different manner as in recently proposed theories (Ozer and Taha, 1986; Freese *et al.*, 1987; Chen and Wu, 1990; Silveira and Waga, 1994; Abdel-Rahman, 1995) of decaying vacuum cosmologies in which a time-dependent cosmological 'constant' is introduced. These theories are also not deducible from a variational principle. Instead of matter energy-momentum conservation, a sum of tensors corresponding to the usual energy-momentum and a 'vacuum energy-momentum' was considered to be conserved. One can regard such conservation in MGR also, as Al-Rawaf and Taha (1996a, b) have shown that MGR can indeed be molded into a model with a variable cosmological 'constant.'

An interesting observational consequence derived from MGR applied to a matter-dominated Robertson–Walker (RW) universe is that it can resolve the conflict between the ages of the oldest stars in our galaxy and that of the universe itself, as derived from the measurement of the Hubble constant from recent observations (Pierce *et al.*, 1994). In fact, as shown by Al-Rawaf and Taha (1996a, b), there is no such problem for $\eta \leq 0.6$ and the present value of the matter density parameter Ω in the range $0.1 \leq \Omega \leq 0.25$ Abdel-Rahman (1997) applied MGR in the radiation-dominated era of the universe and discussed the implications of the nucleosynthesis contraints for the age of the universe. He also showed the consistency of the matter-dominated model developed in the framework of MGR with neoclassical cosmological tests. It was also shown that the baryon asymmetry in the early universe was significantly smaller than at present.

The purpose of the present paper is to examine the very early universe in the framework of MGR. The early universe is taken here to be a thermodynamically open system which can account for particle creation as well as entropy production (De, 1993a). In fact, such an early universe as a thermodynamically open system was considered by Prigogine (1989), who modified the thermodynamic energy conservation law for homogeneous and isotropic universes. An early universe with bulk viscosity also has been considered (De, 1997b). It was shown that the presence of bulk viscous pressure in the early universe can account for the observed specific entropy per baryon in the universe today. Apart from considering the early universe as an open system, the epoch dependence of the masses of elementary particles has been taken into account. The epoch dependence of particle masses, which is significant only in the early period of the evolution of the universe, is in fact a consequence of a Finsler geometric approach to building up the internal symmetry of hadrons considered earlier by De (1991, 1997a). This geometrical approach provides the field equations for the fundamental particles and also the 'dynamics' of hadrons (De, 1986). Here we discuss the implications of MGR in a thermodynamically open early universe with particles having epoch-dependent masses. We will be shown that a 'mild' inflation solution can be found which has no problem with 'turn-on' and 'turn-off.'

We begin in Section 2 with the basic equations of MGR. The equations for the thermodynamically open universe are derived there in the context of MGR. In Section 3, the mild inflation solution is obtained. The specific entropy per baryon is also given. In Section 4, we show how the viscous pressure in the present MGR case can account for the specific entropy per baryon. This can be compared with the value obtained in Section 3 as well as with that in De (1997b). In Section 5, the results of this paper are summed up with some concluding remarks.

2. EARLY UNIVERSE WITH MODIFIED GENERAL RELATIVITY

The gravitational field equations in MGR are (Al-Rawaf and Taha, 1996a)

$$R_{\mu\nu} - \frac{4 - \eta}{6(2 - \eta)} R g_{\mu\nu} = -\frac{8\pi G}{3}\left(1 + \frac{2}{\eta}\right) T_{\mu\nu} \tag{1}$$

where η is a constant in the interval [0, 1]. The value of $\eta = 1$ corresponds to GR.

Here we consider a spatially flat ($k = 0$) RW universe with the metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2) \tag{2}$$

using the natural units $c = \hbar = 1$.

The energy-momentum tensor for the universe as an adiabatic perfect fluid is given by

$$T_{\mu\nu} = -pg_{\mu\nu} + (\rho + p)u_\mu u_\nu \tag{3}$$

where ρ and p are the density and pressure, respectively, and $u_\mu = (1, 0, 0, 0)$ is a unit vector in the time direction.

We get from the field equations (1) the following equations for the MGR cosmology:

$$\frac{\dot{a}^2}{a^2} = \frac{K}{3\eta}[\rho - (1 - \eta)p] \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{K}{6\eta}[\eta\rho + (4 - \eta)p] \tag{5}$$

where

$$K = 8\pi G$$

and the dot represents differentiation with respect to time. The following Bianchi identity follows from (4) and (5):

$$\frac{d}{da}\{a^3[\rho + (\eta - 1)\, p]\} + a^2[(\eta - 1)\, \rho + (5 - 2\eta)p] = 0 \tag{7}$$

or, in differential form,

$$d\{a^3[\rho + (\eta - 1)p]\} + \frac{1}{3}[(\eta - 1)\rho + (5 - 2\eta)p]\, d(a^3) = 0 \tag{8}$$

For $\eta = 1$, we get the usual conservation law for GR,

$$d(\rho a^3) + pd(a^3) = 0 \tag{9}$$

Prigogine (1989) modified this conservation law for the universe considered as a thermodynamically open system. Specifically, equation (9) is modified to the following thermodynamic energy conservation law for a homogeneous and isotropic universe:

$$d(\rho a^3) + pd(a^3) - \frac{h}{n}d(na^3) = dQ \tag{10}$$

where $h = \rho + p$ is the enthalpy per unit volume and $n = N/V$, where N is the number of particles in a given volume V, that is, the comoving volume given by $V = a^3$. For adiabatic transformation we have $dQ = 0$ and we get

$$d(\rho a^3) + pd(a^3) - \frac{h}{n}d(na^3) = 0 \tag{10a}$$

From this conservation law it follows that

$$\dot{\rho} = \frac{\dot{n}}{n}(\rho + p) \tag{11}$$

which replaces the usual Einstein equation (Bianchi identity for homogeneous and isotropic universe), that is,

$$\dot{\rho} = -3H(\rho + p) \tag{12}$$

where H is the Hubble function given by $H = \dot{a}/a$.

Of course, the other Einstein equation corresponding to equation (4) with $\eta = 1$ is still valid for this case of a thermodynamically open universe. This equation is

$$K\rho = 3H^2 \tag{13}$$

One can make an alternative interpretation of the conservation law (10a) or (11) by retaining the usual form of the conservation law (Bianchi identity) with a phenomenological pressure $\hat{p}$ instead of the above true thermodynamic pressure p. That is,

$$d(\rho a^3) = -\hat{p}d(a^3) \tag{14}$$

where the two pressures $\hat{p}$ and p are related by

$$\hat{p} = p + p_c \tag{15}$$

Here, p_c represents a pressure, negative or zero, and corresponds to the creation of particles. In fact, when $p_c = 0$ the creation of particles stops and in this case $\hat{p} = p$. Consequently, the conventional law of conservation holds, or in other words, the usual Einstein equations of GR hold. The pressure p_c is given by

$$p_c = -\frac{\rho + p}{3H}\frac{\dot{S}}{S} \tag{16}$$

where S is the entropy. Prigogine (1989) also showed that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \frac{\dot{n}}{n} + \theta \tag{17}$$

where $\theta = 3H$ is the expansion scalar.

We shall modify the Bianchi identity (8) of MGR for the early universe considered as a thermodynamically open system to account for the creation of particles. This modification is made in the same manner as above, that is, as for GR. For adiabatic transformation ($dQ = 0$) the conservation law (8) changes into the following form:

$$d\{a^3[\rho + (\eta - 1)p]\} + \frac{1}{3}[(\eta - 1)\rho + (5 - 2\eta)p]d(a^3)$$

$$- \frac{h}{n} d(na^3) = 0 \tag{18}$$

Equation (4) remains valid for this case of a thermodynamically open early universe and becomes the usual Einstein equation (13) for GR if $\eta = 1$. From (18), it follows that

$$\dot{\rho} + (\eta - 1)\,\dot{p} - (p + \rho)\left[\frac{\dot{n}}{n} - (\eta - 1)\frac{\theta}{3}\right] = 0 \tag{19}$$

Now, in the particle production era of the very early universe the energy density $\rho = \rho_m + \rho_\gamma$ should be dominated by the matter density ρ_m, that is, $\rho_m >> \rho_\gamma$, the radiation energy density. Consequently, we can take

$$\rho = mn \tag{20}$$

where m is the particle mass. Here, for simplicity, the masses of all types of particles created are assumed to be the same. The mass of the particle has been taken to be epoch dependent. In fact, in the previous consideration of hadronic matter extension in microlocal space-time regarded as an anisotropic Finsler space (De, 1991, 1997a), the epoch dependence of masses of elementary particles appeared as an important consequence. For the universe described by the background metric of RW type this epoch dependence of masses plays a significant role in the very early stage of its evolution. The actual relation obtained there that gives rise to the mass of the particle at an epoch time t is given by

$$m = \bar{m}\,[1 + 2\alpha H(t)] \tag{21}$$

where $\alpha = 0.26 \times 10^{-23}$ sec and $\bar{m}$ represents the 'inherent' mass of the particle. This inherent mass is equal to the present mass of the particle with a very high degree of accuracy. For massless particles (that is, for particles with no inherent mass) the corresponding relation is

$$m = 2\alpha\hat{m}H(t) \tag{22}$$

where $\hat{m}$ is the mass of the particle at the epoch time $t = \alpha$.

Now, equations (4) and (19)–(21) together with the equation of state

$$p = F(\rho) \equiv \rho f(\theta) \tag{23}$$

are the governing equations that describe the early evolution of the thermodynamically open universe with MGR.

Using (20) and the equation of state (23), we have from (19)

$$(\eta - 2)f(\theta)\frac{\dot{\rho}}{\rho} + (\eta - 1)f'(\theta)\dot{\theta} + [1 + f(\theta)]\left[\frac{\dot{m}}{m} + (\eta - 1)\frac{\theta}{3}\right] = 0 \tag{24}$$

Again, from (21) it follows that

$$\frac{\dot{m}}{m} = \frac{2\alpha\dot{\theta}}{3 + 2\alpha\theta} \tag{25}$$

and from (4), by using (23), we get

$$\rho = \frac{\eta\theta^2}{3K[1 + (\eta - 1)f(\theta)]} \tag{26}$$

Consequently, we have

$$\frac{\dot{\rho}}{\rho} = \frac{2\dot{\theta}}{\theta} - \frac{(\eta - 1)f'(\theta)\dot{\theta}}{1 + (\eta - 1)f(\theta)} \tag{27}$$

With the use of equations (25) and (27) we obtain from (24) the following equation:

$$(\eta - 2)f(\theta)\left[\frac{2\dot{\theta}}{\theta} - \frac{(\eta - 1)f'(\theta)\dot{\theta}}{1 + (\eta - 1)f(\theta)}\right] + (\eta - 1)f'(\theta)\dot{\theta} + [1 + f(\theta)]\left[(\eta - 1)\frac{\theta}{3} + \frac{2\alpha\dot{\theta}}{3 + 2\alpha\theta}\right] = 0 \tag{28}$$

Again, as pointed out earlier, the creation era of the very early universe is matter-dominated, and relation (20) holds for the energy density; consequently it follows that

$$\rho a^3 = Nm \tag{29}$$

By using (21) and (26), we get the number of particles in a comoving volume $V = a^3$ in terms of the expansion scalar θ as follows:

$$N = \frac{\eta\theta^2 a^3}{K\bar{m}(3 + 2\alpha\theta)[1 + (\eta - 1)f(\theta)]} \tag{30}$$

Equations (28) and (30) describe the 'creation-era' of the early universe in the MGR formulation.

3. MILD INFLATION IN THE EARLY UNIVERSE

Let us first consider the GR case in the thermodynamically open early universe. The governing equations for this case are (11), (13), and (21) together with the relation (20) for energy density, since the universe in its early era was matter-dominated. From these equations, the following equations can be deduced (De, 1993a) if the equation of state (23) is incorporated:

$$\frac{\dot{\rho}}{\rho} = \frac{2\dot{\theta}}{\theta} = \frac{\dot{n}}{n}[1 + f(\theta)] = \frac{\dot{m}}{m}\frac{1 + f(\theta)}{f(\theta)} \qquad \text{with} \quad \frac{\dot{m}}{m} = \frac{2\alpha\dot{\theta}}{3 + 2\alpha\theta} \tag{31}$$

These equations have a trivial solution

$$\dot{\theta} = \dot{\rho} = \dot{n} = \dot{m} = 0$$

which represents the usual inflation. Apart from this trivial solution, these equations cannot determine the expansion scalar θ. On the other hand, if $\dot{\theta} \neq 0$, one can determine the equation of state, that is, the function $f(\theta)$ can be specified. It is

$$f(\theta) = \frac{\alpha\theta}{3 + \alpha\theta} \tag{32}$$

The expansion scalar θ, however, may be specified with the requirement that $\hat{p} = 0$ in equation (14), as this era of the universe is matter-dominated. In fact, this phenomenological zero pressure ushers in a matter-dominated RW universe with the scale factor $a(t) \propto t^{2/3}$. Note that for $\hat{p} = 0$, $p_c = -p = -f(\theta)\rho$. This negative pressure p_c is responsible for particle creation in the era $t << \alpha$. On the other hand, as t increases, $f(\theta)$ decreases and in fact, $f(\theta) \to 0$ as t becomes large. It was argued in De (1993a) that the universe around the epoch time $t = \alpha$, regarded as a transition epoch, becomes a radiation-dominated RW universe, the usual universe according to the standard cosmology and with no particle creation. In fact, the particles at this transition epoch become relativistic and contribute to the radiation-energy density, that is, $\rho = \rho_\gamma$, $f(\theta) = 1/3$, $p_c = 0$, and consequently $a(t) \propto t^{1/2}$.

Now, for MGR we rewrite equation (28) in the following form:

$$\begin{aligned} 2\dot{\theta}&\left\{\frac{\alpha[1 + f(\theta)]}{3 + 2\alpha\theta} - \frac{f(\theta)}{\theta}\right\} \\ &+ (\eta - 1)\left\{f(\theta)\dot{\theta}\left[\frac{2}{\theta} - \frac{(\eta - 2)f'(\theta)}{1 + (\eta - 1)f(\theta)}\right]\right. \\ &\left.+ f'(\theta)\dot{\theta} + \frac{\theta}{3}[1 + f(\theta)]\right\} = 0 \end{aligned} \tag{33}$$

Obviously, for $\eta = 1$, that is, for GR, we arrive at the same results as stated above, that is, either $\dot{\theta} = 0$ or the specified equation of state $p = f(\theta)\rho$, with $f(\theta)$ given by (32). The usual inflationary solution is represented by the trivial case $\dot{\theta} = 0$. Of course, the actual physical conditions that might determine the turning on and turning off of this inflationary paradigm are still obscure, although it is now believed that they are similar to a symmetry-breaking phase transition. The dominance of the vacuum energy field may be responsible for triggering a symmetry breaking at the GUT energy scale. On the other hand, the switching off of the inflationary phase is assumed to occur due to a second phase transition in which the symmetry of the weak nuclear force and electrostatic force is broken.

From equation (33) it is clear that one can determine in principle the expansion scalar θ and consequently the scale factor $a(t)$ if one knows $f(\theta)$, that is, the equation of state. Conversely, if one knows the character of the expansion of the universe [either $a(t)$ or $\theta(t)$], the equation of state can be obtained from this equation. It is also clear that if $\eta = 1$ or if $\eta \to 1$, the equation of state can be specified and is given by (32). Thus, one may suppose the same functional dependence $f(\theta)$ on the expansion scalar for all values of η in [0, 1]. This provides us the following equation for θ which decides the nature of the early universe:

$$f(\theta)\dot{\theta}\left[\frac{2}{\theta} - \frac{(\eta - 2)f'(\theta)}{1 + (\eta - 1)f(\theta)}\right] + f'(\theta)\dot{\theta} + \frac{\theta}{3}[1 + f(\theta)] = 0 \qquad (34)$$

This equation is valid for all values of η which satisfy $0 \leq \eta < 1$. Since it holds for $\eta \to 1$, one can take it to be valid for the case $\eta = 1$, that is, for GR. Of course, previously we have taken $\hat{p} = 0$ for this matter-dominated creation era of the early universe to find the expansion scalar θ. Such an assumption of zero phenomenological pressure is not necessary. Equation (34) gives the expansion scalar for all values of η in the range [0, 1]. Again, by using the expression for $f(\theta)$ from (32) in (34) we arrive at the following equation:

$$\dot{\theta}\left[\frac{3}{\theta} - \frac{4\alpha}{3 + 2\alpha\theta} - \frac{\alpha}{(1 - \eta)(3 + \alpha\theta)} + \frac{\eta^2\alpha}{(1 - \eta)(3 + \eta\alpha\theta)}\right] + \frac{1}{\alpha} = 0 \qquad \text{for} \quad \eta \neq 1 \qquad (35)$$

and

$$\frac{\dot{\theta}}{\theta}\left[\frac{2\alpha}{3 + 2\alpha\theta} + \frac{3\alpha}{(3 + \alpha\theta)^2}\right] + \frac{1}{3} = 0 \qquad \text{for} \quad \eta = 1 \qquad (36)$$

One can find the solution of equation (35) by adjusting the constant of integration suitably. In fact, here the integration constant has been taken to ensure that $\alpha\theta$ is very large for epoch times $t << \alpha$. Thus, for $t < \alpha$ we can have

$$\frac{4(\alpha\theta)^3(3+\eta\alpha\theta)^{\eta/(1-\eta)}}{(3+2\alpha\theta)^2(3+\alpha\theta)^{1/(1-\eta)}} = (\eta^{\eta/(1-\eta)})e^{-t/\alpha} \tag{37}$$

For $t << \alpha$, for which $\alpha\theta$ is very large, we have from (37)

$$\left[1 - \frac{3}{\alpha\theta} - \frac{3}{(1-\eta)\alpha\theta}\right]^{1+3/(1-\eta)\alpha\theta} = 1 - \frac{t}{\alpha}$$

or

$$1 - \frac{3}{\alpha\theta} - \frac{3}{(1-\eta)\alpha\theta} + \frac{3}{(1-\eta)\alpha\theta} = 1 - \frac{t}{\alpha}$$

Therefore,

$$\theta = 3/t \qquad \text{or} \qquad H = 1/t \tag{38}$$

This gives the scale factor

$$a(t) \propto t \tag{39}$$

This scale factor indicates a ‘mild inflation’ in the very early stage of evolution of the universe. For $\eta = 1$, we get from (36) the solution for θ. By adjusting the constant of integration in the same manner as above, we get the following relation, which determines θ for $t < \alpha$:

$$\frac{4(\alpha\theta)^3}{(3+2\alpha\theta)^2(3+\alpha\theta)} \exp\left(\frac{3}{3+\alpha\theta}\right) = \exp\left(-\frac{t}{\alpha}\right) \tag{40}$$

Note that we can also arrive at the above solution from (37) by making $\eta \to 1$. From (40), we can find θ for $t << \alpha$. In fact, for this case of $\eta = 1$, that is, for GR we get the same expansion scalar θ and scale factor $a(t)$ as in (38) and (39), respectively. Thus, for all values of the parameter η in the range $0 \leq \eta \leq 1$ we have the mild inflation phase before the time α.

Now, from the conservation law (18) in MGR for the early universe as a thermodynamically open system with adiabatic transformation, it is obvious that the pressure p_c is given by

$$p_c = -\frac{(p+\rho)(d/dt)(na^3)}{n(d/dt)(a^3)} \tag{41}$$

[cf. equations (10a), (16), and (17)].

Using (20), (25), and (27) we have from (41)

$$p_c = -\frac{\rho[1+f(\theta)]}{\theta}\left\{\dot{\theta}\left[\frac{2}{\theta} - \frac{(\eta-1)f'(\theta)}{1+(\eta-1)f(\theta)} - \frac{2\alpha}{3+2\alpha\theta}\right] + \theta\right\} \quad (42)$$

Again, from equation (34) we find

$$\frac{[1+f(\theta)]f'(\theta)\dot{\theta}}{1+(\eta-1)f(\theta)} = -\left\{\frac{2f(\theta)\dot{\theta}}{\theta} + \frac{\theta}{3}[1+f(\theta)]\right\} \quad (43)$$

Using this, we have from (42) the following expression for the pressure p_c:

$$p_c = -\frac{\rho}{\theta}\left\{[1+f(\theta)]\left[\frac{2\dot{\theta}}{\theta}\frac{3+\alpha\theta}{3+2\alpha\theta} + \theta + (\eta-1)\frac{\theta}{3}\right] + (\eta-1)\frac{2f(\theta)\dot{\theta}}{\theta}\right\} \quad (44)$$

Finally, by using the expression for $f(\theta)$ from (32), we obtain

$$p_c = -\frac{\rho}{3+\alpha\theta}\left[\frac{2\dot{\theta}}{\theta^2}(3+\eta\alpha\theta) + \frac{\eta+2}{3}(3+2\alpha\theta)\right] \quad (45)$$

It is evident from this equation that for $\theta = 3/t$, that is, for the era $t << \alpha$, the pressure $p_c = -(4/3)\rho$. Due to this negative pressure the particle creation continues for time $t << \alpha$. As the time t comes closer to the epoch time α, the function $f(\theta)$ changes from almost equal to one to a value less than unity. In fact, it is evident from (37) and (40) that $\alpha\theta$ becomes close to zero as the time t becomes larger than α. To find the behavior of the pressure p_c with respect to the changes in $\alpha\theta$, we put equation (35) into (45) to get

$$p_c = -\rho\left\{\frac{1}{3}(\eta+2)\frac{3+2\alpha\theta}{3+\alpha\theta} - \frac{2}{\alpha\theta}\frac{3+\eta\alpha\theta}{3+\alpha\theta} \times\left[3 - \frac{4\alpha\theta}{3+2\alpha\theta} - \frac{\alpha\theta}{(1-\eta)(3+\alpha\theta)} + \frac{\eta^2\alpha\theta}{(1-\eta)(3+\eta\alpha\theta)}\right]^{-1}\right\} \quad (46)$$

For very small $\alpha\theta$, p_c becomes positive and consequently there should be no creation of particles due to this pressure. Even for $\alpha\theta = O(1)$, for example, if $\alpha\theta = 0.9$, the pressure $p_c \geq 0$ for values of η either near zero or near unity. For $\alpha\theta = 0.3$, $p_c \geq 0$ for all values of η in the range $0 \leq \eta \leq 1$. Again, it is apparent from (37) and (40) that when $\alpha\theta$ becomes of the order of unity, the time t is around the time α. Thus, for all values of η in [0, 1], we have $p_c \geq 0$ around the epoch time α and hence the creation of particles brought about by the pressure p_c must stop around that time. Consequently, the usual cosmology either with MGR or GR (the $\eta = 1$ case) must set in after the epoch time α. In fact, the universe then becomes a radiation-dominated RW universe since it is no longer thermodynamically open and matter-dominated with particle creation. Of course, one has a different situation for $\eta \neq 1$ or $\eta = 1$ (the case of GR), although, as pointed out earlier, the universe described in the MGR framework has no observational discrepancy from the present state of the universe (Abdel-Rahman, 1997).

An important aspect of this 'mild inflation' is that in this case there are no problems with its turning on and turning off. As seen above, this inflation is automatically turned off at the time scale α. On the other hand, is was turned on at the Planck-order time scale as discussed in a previous paper (De, 1993b). It is shown there that very massive particles (more than 50 times the Planck mass m_{Pl}) might be created quantum mechanically in an initially 'anisotropically perturbed' Minkowski space-time for a duration of the Planck-order time. These very massive particles can make the Minkowski space-time unstable (Nardone, 1989) and thrust it into an expansion phase, the beginning phase of the expanding universe at this Planck-order time. De (1993b) also considered the quantum creation of particles in the curved space-time of this very early expanding phase of the universe. Such considerations can indeed give results for the radiation energy density, temperature of the universe, baryon number, etc., at the transition epoch α in agreement with observation. The 'particle creation era' of the expanding phase of the universe which was turned on from the Minkowski space must be a thermodynamically open system, as considered here. The nature of the expansion has been found here to be a mild inflation. Thus, the universe is switched into a mild inflationary stage from the Planck-order time up to an epoch time α, when it automatically, undergoes a transition into a radiation-dominated RW universe. It is an interesting fact that the highly massive particles created in this very early period (at the Planck-order time scale) may be primordial black holes or even known elementary particles such as muons, electrons, and massive neutrinos, whose masses are in fact on the order of more than 50 times the Planck mass owing to the epoch dependence of particle masses as mentioned above [see equations (21) and (22)].

In the present consideration of mild inflation, the number of particles created during this phase can be estimated. If $N(\hat{t})$ is the number of particles in the comoving volume a^3 at the time $\hat{t}$ (Planck-order time) at which inflation is turned on and $N(\alpha')$ is that at the epoch $\alpha' < \alpha$, the time at which it is turned off, then we obtain the following relation from equation (30):

$$\frac{N(\alpha')}{N(\hat{t})} = \frac{\theta^2(\alpha')a^3(\alpha')[3 + 2\alpha\theta(\hat{t})]\,[1 + (\eta - 1)f(\theta(\hat{t}))]}{\theta^2(\hat{t})a^3(\hat{t})[3 + 2\alpha\theta(\alpha')]\,[1 + (\eta - 1)f(\theta(\alpha'))]} \tag{47}$$

Here, α' is the time up to which the universe is matter-dominated, that is, the relation (20) for the energy density is valid. The time interval from

α' to α can be regarded as the transition era from this matter-dominated, thermodynamically open universe to the radiation-dominated RW universe. From (47) we have, by using the expression for the expansion scalar from (38),

$$\frac{N(\alpha')}{N(\hat{t})} \simeq \left(\frac{\alpha'}{\hat{t}}\right)^2 \tag{48}$$

The time $\hat{t}$ represents the epoch time at which the masses of the particles are 54 times the Planck mass. It can be found that $\hat{t} \simeq 0.05 t_{\mathrm{Pl}}$. If we take $\alpha' = 0.1\alpha$, we find an estimate of the number of particles at the epoch α' in relation to that at the epoch $\hat{t}$:

$$N(\alpha') \simeq 10^{40} N(\hat{t}) \tag{49}$$

Thus, during the phase of mild inflation the number of particles increases by a factor of 10^{40}. After this phase the nature of the expansion scalar changes and the particle creation stops. In fact, the phenomenological pressure $\hat{p}$ in the MGR framework is given by

$$\hat{p} = -\rho\left[\left(1 + \frac{2\dot{\theta}}{\theta^2}\right) + (\eta - 1)\left(\frac{1}{3} + \frac{2f(\theta)}{1 + f(\theta)}\frac{\dot{\theta}}{\theta^2}\right)\right] \tag{50}$$

With this pressure the conservation law (Bianchi identity) takes the usual form (14). When the creation of particles due to the pressure p_c stops around $t = \alpha$ at the onset of the radiation-dominated RW era with $\theta = 3/2t$, the pressure $\hat{p}$ changes and, indeed $\hat{p} = \rho/3$ at $t = \alpha$ together with $f(\theta) = 1/3$. For this era, it can be shown that (De, 1993a)

$$\rho_\gamma/\rho_m = \gamma^2 - 1 \tag{51}$$

where

$$\gamma = (1 - \langle v^2\rangle/c^2)^{-1/2} \tag{52}$$

$\langle v^2\rangle$ is the mean square velocity of the particles. In the transition to the radiation era the particles become relativistic and consequently the value of γ increases. It was argued in De (1993a) that the part of the energy density $\gamma m n_m$ (n_m being the number density of massive particles) due to the massive particles contributes to the radiation energy density ρ_γ, and due to the fact that γm for large γ at the epoch α becomes the relativistic mass-energy it must be of the order of E_γ at that epoch, that is,

$$E_\gamma \simeq \gamma_\alpha m \tag{53}$$

where γ_α is the value of γ at $t = \alpha$ and E_γ is the energy per photon. This relation (53) determines the value of γ_α from the known values of the particle mass and E_γ at the epoch $t = \alpha$, since the standard cosmology follows. The value of E_γ at $t = \alpha$ is 10^{22} cm^{-1}, and $m \simeq 10^{13}$ cm^{-1} for muons (taking them as representative particles) in units $\hbar = c = K = 8\pi G = 1$. With these values for E_γ and m we find $\gamma_\alpha \simeq 10^9$. Consequently, from (51) it follows that, at $t = \alpha$,

$$\frac{E_\gamma\, n_\gamma}{m n_m} \simeq \gamma_\alpha^2$$

and using (53), we arrive at the ratio

$$\frac{n_\gamma}{n_m} \simeq \gamma_\alpha \simeq 10^9 \tag{54}$$

This ratio of the photon to particle number remains constant after the transition from the creation to the radiation era in the GR framework because the particle creation has ceased. In the GR framework, the present value of the specific entropy σ_0 per baryon is

$$\sigma_0 = 3.7 \frac{\rho_{\gamma 0}}{\rho_{m0}} \frac{m_b}{T_0} \tag{55}$$

where $\rho_{\gamma 0}$, ρ_{m0}, and T_0 refer to the present values of radiation density, matter density, and temperature of the universe, respectively. m_b is the mass of a baryon. We can use the following adiabatic constant (since, after the epoch $t = \alpha$, the usual cosmology follows and we are considering the GR case)

$$\rho_\gamma/\rho_m T = \text{const} = \rho_{\gamma 0}/\rho_{m0} T_0 \tag{56}$$

From this, we find at $t = \alpha$, using (51),

$$\rho_\gamma/\rho_m = \rho_{\gamma 0} T/\rho_{m0}\, T_0 = \gamma_\alpha^2 - 1 \simeq 10^{18} \tag{57}$$

The temperature T at the epoch α can be computed using the Einstein equation (13) and the usual relation

$$\rho_\gamma = (\pi^2/30) N_{\text{eff}}\, T^4 \tag{58}$$

where N_{eff} is the effective number of relativistic particle spin degrees of freedom. We find $T \simeq 10^{22}$ cm^{-1}. Thus, we get

$$\sigma_0 = 3.7 \frac{m_b}{T} \times 10^{18} = 1.76 \times 10^{10} \tag{59}$$

(using the proton mass as the representative mass for the baryons). This value of σ_0 is in agreement with observation. In the MGR framework, however, the specific entropy per baryon does not remain constant after the epoch α

when particle creation due to the pressure p_c stops, but creation continues in the subsequent era governed by standard cosmology in MGR (because of the nonconservation of matter). In fact, it is shown by Abdel-Rahman (1997) that the specific entropy per baryon remains proportional to $T^{1-\eta}$ in the standard cosmology, that is, the baryon asymmetry was much smaller in the early universe than at present. Thus,

$$\frac{\sigma_0}{\sigma_\alpha} = \left(\frac{T_0}{T_\alpha}\right)^{1-\eta} \tag{60}$$

where σ_α and T_α are the corresponding quantities at the epoch time α. However, for a value of η in the range $0.9 < \eta \leq 1$, the present value of the specific entropy σ_0 per baryon is $\geq 10^8$, which is still within the observational limits. In the following the production of specific entropy per baryon as an effect of bulk viscosity in the early universe will be discussed in the framework of MGR.

4. BULK VISCOSITY IN THE EARLY UNIVERSE UNDER MGR

In a previous paper (De, 1997b) the effect of bulk viscous pressure in the early universe was considered in the framework of GR. Here we shall study its effect in the MGR formulation of the early universe as a thermodynamically open system. We shall consider bulk viscosity under equilibrium thermodynamics. Its role in the matter creation model has been justified in Sudharsan and Johri (1994). In this case, the stress-energy tensor is given by

$$T_{\mu\nu} = (\rho + p + p_c - \zeta\theta)u_\mu u_\nu - (p + p_c - \zeta\theta)g_{\mu\nu} \tag{61}$$

where ζ, the coefficient of bulk viscosity, is in general a function of time. Consequently, the Einstein equation (4) and the conservation law (18) for the thermodynamically open universe are modified into the following forms:

$$\theta^2 = \frac{3K}{\eta}[\rho - (1 - \eta)(p - \zeta\theta)] \tag{62}$$

$$\begin{aligned} d\{a^3[\rho - (1 - \eta)(p - \zeta\theta)]\} + \tfrac{1}{3}[(\eta - 1)\rho \\ + (5 - 2\eta)(p - \zeta\theta)]d(a^3) - \frac{h}{n}\, d(na^3) = 0 \end{aligned} \tag{63}$$

From equations (62) and (63), we arrive at

$$\begin{aligned} \frac{\eta}{3K}(2\dot{\theta} + \theta^2)\theta + \frac{\theta}{3}[(\eta - 1)\rho \\ + (5 - 2\eta)(p - \zeta\theta)] - (p + \rho)\left(\frac{\dot{n}}{n} + \theta\right) = 0 \end{aligned} \tag{64}$$

When the particle creation in the open universe stops, $\dot{N}/N = 0$, that is, $(\dot{n}/n) + \theta = 0$ and the universe goes through the usual radiation-dominated era (of course, in the framework of MGR and with bulk viscosity) for which $\theta = 3/2t$, and consequently $\dot{\theta} + 2\theta^2/3 = 0$, and we have, from (64),

$$\eta\theta^2/3K = (\eta - 1)\rho + (5 - 2\eta)(p - \zeta\theta) \tag{65}$$

Now, using equation (62) and the equation of state (23) we find, from (65),

$$\zeta(t) = (1/\theta)[f(\theta) - \tfrac{1}{3}]\rho \tag{66}$$

Again from (62), we get

$$\rho = \frac{\eta\theta^2/3K - (1 - \eta)\zeta\theta}{1 - (1 - \eta)f(\theta)} \tag{67}$$

From equations (66) and (67), by eliminating ρ, we finally arrive at the following expression for the coefficient of bulk viscosity:

$$\zeta(t) = \frac{\eta\theta}{(2 + \eta)K}\left[f(\theta) - \frac{1}{3}\right] \tag{68}$$

Now, the entropy equation in this case is given by

$$T\frac{\dot{S}}{V} = \zeta\theta^2 + \frac{TS}{V}\frac{\dot{N}}{N} \tag{69}$$

where $V = a^3(t)$ is the comoving volume. This equation can be written in terms of specific entropy per particle σ: given as

$$\dot{\sigma} = \zeta\theta^2/nT \tag{70}$$

It is evident that $\sigma = \text{const}$ when $\zeta = 0$. Note that when particle creation stops, $\dot{N} = 0$, the relation (70) remains valid. When $f(\theta) = 1/3$ at the epoch α at the onset of the radiation-dominated RW era, $\zeta(t)$ becomes zero and σ remains constant afterward. But in the transition era before that epoch time, $f(\theta) > 1/3$ and production of specific entropy per particle is possible. In fact, in the GR formulation of the early universe as a thermodynamically open universe with bulk visosity it was shown in De (1997b) that for the equation of state $p = f(\theta)\rho$, where $f(\theta)$ is given by (32), the coefficient of bulk viscosity must vanish during the 'particle production' (matter-dominated) RW era until the transition epoch. In this era, creation of particles and entropy production continues with constant specific entropy per particle σ. In the present MGR formulation we consider the case of nonzero $\zeta(t)$ in the transition epoch just before the epoch time α. The equation for σ follows from (68) and (70):

$$\dot{\sigma} = \frac{\eta\theta^3}{(2 + \eta)KTn}\left[f(\theta) - \frac{1}{3}\right] \tag{71}$$

Now, we can use the standard cosmological invariant (which is valid from this transition period to the present era) given by

$$RT = R_0T_0 = C = 1.18 \times 10^{29}\, u \; (1 < u < 1.8) \tag{72}$$

where R_0 and T_0 are the present scale factor and the temperature of the universe, respectively. As particle creation stops, $N = nR^3$ remains constant afterward. Then, from (71) we have

$$\dot{\sigma} = \frac{\eta\theta^3C^3}{(2 + \eta)KNT^4}\left[f(\theta) - \frac{1}{3}\right] \tag{73}$$

An estimate of specific entropy produced per particle during the short transition period specified as $(\alpha - \Delta\alpha, \alpha)$ can be obtained. This estimate for the period $\Delta\alpha$ just before α is given by

$$\Delta\sigma = \frac{\eta\theta_\tau^3C^3}{(2 + \eta)KNT_\tau^4}\left[f(\theta_\tau) - \frac{1}{3}\right]\Delta\alpha \tag{74}$$

where the subscript τ represents the values for the corresponding quantities at some intermediate epoch τ of the transition period given by $\tau = \alpha - r\,\Delta\alpha$, $0 < r < 1$. Now, from (72) it also follows that

$$T_\alpha/T_\tau = (\tau/\alpha)^{1/2} \tag{75}$$

In natural units, $T_\alpha \simeq 10^{22}\ \text{cm}^{-1}$. Also, $K = 8\pi G = 1/m_{\text{Pl}}^2 = t_{\text{Pl}}^2$ in these units, m_{Pl} and t_{Pl} being, respectively, the Planck mass and time. Putting $\Delta\alpha/\alpha = q$, we have from (74), by using (32), (72), and (75),

$$\begin{aligned}\Delta\sigma &= \left(\frac{\alpha}{2\tau + \alpha} - \frac{1}{3}\right)\frac{27\eta C^3\,\Delta\alpha}{8(\eta + 2)\tau t_{\text{Pl}}^2\,NT_\alpha^4\alpha^2}\\ &= \left(\frac{1}{3 - 2rq} - \frac{1}{3}\right)\frac{27\eta C^3 q}{8(\eta + 2)Nt_{\text{Pl}}^2T_\alpha^4\,\alpha^2(1 - rq)}\\ &= 2.09 \times 10^{12}\left[\frac{\eta q}{(\eta + 2)(1 - rq)}\left(\frac{1}{3 - 2rq} - \frac{1}{3}\right)\right]\end{aligned} \tag{76}$$

where we have used the total particle (baryon) number of the present universe, given by

$$N_b = 2.45 \times 10^{78}u^3 \qquad (1 < u < 1.8) \tag{77}$$

as calculated in De (1993b). There the matter creation was considered quantum mechanically for the initial anisotropic fluctuation of Minkowski space-time and subsequently in the matter-dominated RW era up to the epoch time α. With the specification of r and q (which specify the period of transition) as $r = 0.5$ and $q = 1/3$ we find an estimate of the specific entropy produced per particle as

$$\Delta\sigma \simeq 3.48 \times 10^{10}\eta/(\eta + 2) \tag{78}$$

As pointed out above, the effect of viscous pressure vanishes after the epoch α when $f(\theta) = 1/3$, and in the GR case, for which $\eta = 1$, the specific entropy per particle is, from (78),

$$\Delta\sigma \simeq 1.16 \times 10^{10} \tag{79}$$

which remains constant afterward until the present state of the universe. This value is also very close to that in (59). If the transition period is a bit shorter, say $q = 1/4$, the specific entropy per particle is (in the GR case) 6×10^9. The interesting point is that this observable quantity of the present universe was produced in the transition period just before the epoch time α. For the MGR case, however, as pointed out earlier, this quantity decreases. Only for a value of η in the range $0.9 < \eta \leq 1$ is its present value within the observational limits. For a smaller value of η, the transition period might have been much longer to account for a larger production of specific entropy per particle. This can be calculated from (73) using (75) and (77), and for the transition period from the epoch τ to the epoch α, we have

$$\begin{aligned}
\sigma &= \frac{\eta C^3}{(2+\eta)KN}\int_\tau^\alpha \frac{\theta^3}{T^4}\left[f(\theta) - \frac{1}{3}\right]dt \\
&= 2.09 \times 10^{12}\frac{\eta}{2+\eta}\int_\tau^\alpha \frac{1}{t}\left(\frac{\alpha}{2t+\alpha} - \frac{1}{3}\right)dt \\
&= 2.09 \times 10^{12}\frac{\eta}{2+\eta}\left(\frac{2}{3}\ln\frac{\alpha}{\tau} - \ln\frac{3\alpha}{2\tau+\alpha}\right)
\end{aligned} \tag{80}$$

The present value of the specific entropy per particle is given by

$$\sigma_0 = 2.09 \times 10^{12}\frac{\eta}{2+\eta}\left(\frac{2}{3}\ln\frac{\alpha}{\tau} - \ln\frac{3\alpha}{2\tau+\alpha}\right)\left(\frac{T_0}{T_\alpha}\right)^{1-\eta} \tag{81}$$

However, a much smaller value of η from the range $0.9 < \eta \leq 1$ cannot account for the present value σ_0 within its observational limits. For example, for a transition period of $\tau = 10^{-26}$ sec, η should be equal to 0.8 to have $\sigma_0 \simeq 10^8$, the observational lower limit. Even a greater transition period cannot give rise to a value of $\eta \leq 0.75$ if $\sigma_0 \gtrsim 10^8$. Therefore, from this observational fact, it is evident that the parameter η should lie in the range $0.75 < \eta \leq 1$.

It is interesting to note that in the GR formulation the usual inflation for which $\dot{\theta} = 0$ cannot produce specific entropy per particle. In fact, for $\eta = 1$, we have from (62) and (64)

$$\theta^2 = 3K\rho \tag{82}$$

$$\frac{1}{3K}(2\dot{\theta} + \theta^2)\theta + \theta(p - \zeta\theta) - (p + \rho)\left(\frac{\dot{n}}{n} + \theta\right) = 0$$

From these equations, with the use of the equation of state (23) together with equations (20) and (25), we arrive at the following expression for the coefficient of bulk viscosity:

$$\zeta(t) = \frac{2\dot{\theta}}{3K\theta(3 + 2\alpha\theta)}[\alpha\theta - (3 + \alpha\theta)f(\theta)] \tag{83}$$

Clearly, $\zeta(t) = 0$ for $\dot{\theta} = 0$. Consequently, from (70), it follows that $\dot{\sigma} = 0$. Thus, for the usual inflation σ remains constant.

5. SUMMARY AND CONCLUDING REMARKS

In this paper we have investigated the very early universe as a thermodynamically open system in the framework of modified general relativity (MGR). This MGR was originally proposed by Rastall (1972) and later by Al-Rawaf and Taha (1996a,b). MGR contains a new fundamental constant η, which may be considered as a characteristic of the non-Newtonian regime, besides the usual gravitational constant G. The parameter η lies in the range $0 \leq \eta \leq 1$. Energy conservation is not imposed in this theory as a fundamental assumption. General relativity (GR) is included here holding for $\eta = 1$. The early universe considered throughout this article is a spatially flat RW universe and is a thermodynamically open system which allows irreversible matter creation from the gravitational field, as originally proposed by Prigogine *et al.* (1988, 1989). In fact, the thermodynamic conservation law has been modified appropriately for an open system under adiabatic transformations. A negative pressure p_C is responsible for the matter creation which acts as a source of internal energy. Again, the irreversibility of the process of particle creation from the gravitational energy is assured from the second law of thermodynamics. Thus, in this formalism the second law of thermodynamics is incorporated in a more meaningful way into the evolution of the system.

The governing equations that describe the early stage of the evolution of the universe, the open system under adiabatic transformation, have been derived here in the MGR framework. With the incorporation of the epoch dependence of particle mass and the equation of state it has been possible to obtain the equation for the expansion scalar θ describing the creation era of the early universe. The epoch dependence of particle mass is a result of the previous investigations (De, 1991, 1997a) on 'extended' hadrons in the microlocal space-time, which is regarded as an anisotropic Finslerian space-time. From the equation of the expansion scalar it is possible to find the nature of the function $f(\theta)$ in the equation of state and also a 'mild inflation' solution. During this period of inflation particles were created. It is shown that the number of particles increased by a factor of 10^{40}. In the case $\eta = 1$ (that is, for GR) one can regain the governing equations obtained in previous work (De, 1993a). Of course, the present equations have the advantage that they can give a specific equation for the expansion scalar. In the previous considerations it was obtained from the assumption of a vanishing phenomenological pressure $\hat{p}$. Of course, there as well as in the present consideration for the GR case ($\eta = 1$) a trivial solution exists and it is the usual inflation. But in MGR with $\eta \neq 1$ no such solution appears. The most important aspect regarding the 'mild inflation' solution is that there are no turn-on and turn-off problems which are generally prevalent in the case of usual inflation unless extra physical conditions, such as symmetry-breaking phase transitions, are assumed to be responsible for those transitions. On the contrary, for the mild inflation the switch-off of this phase occurs automatically on the time scale α ($\simeq 10^{-23}$ sec) when the universe became radiation-dominated RW era with usual (that is, not thermodynamically open) MGR cosmology and, of course, if $\eta = 1$, with GR cosmology. The switching on of the mild inflationary phase happens at a Planck-order time when quantum creation of very massive particles (of masses $> 53.3 m_{Pl}$) in an anisotropically perturbed Minkowski space-time (De, 1993b) makes that space-time unstable, as shown by Nardone (1989), and ushers in an expansion phase which, in the present case, is a mild inflation. This Planck-order 'turning-on' time marks the beginning of the expanding universe. The highly massive particles created might be either primordial black holes or known elementary particles, such as, muons, electrons, neutrinos, etc. Because the mass of an elementary particle is dependent on the epoch, the masses of the created particles (leptons) at the Planck-order epoch are in fact as high as more than 50 times the Planck mass.

The specific entropy per baryon was calculated for both the GR ($\eta = 1$) and MGR ($\eta \neq 1$) cases. It was obtained from the consideration of the ratio of matter and radiation energy densities at the transition epoch α after which the mild inflationary stage turns into a radiation-dominated usual RW (with GR or MGR) universe. Although the calculated value of the specific entropy σ per baryon for the GR case is in agreement with observations, for the MGR case it is within the observational limits only for values of η in the range $0.9 < \eta < 1$. The production of specific entropy per particle was also considered in the MGR framework with the introduction of bulk viscous pressure. The fact that bulk viscosity can account for the production of σ was long ago conceived by Zel'dovich (1970) and subsequently considered by others (see Desikan, 1997, for references). The present consideration of specific entropy per particle in the early universe with MGR gives rise to a value within the observational limits for η in the range $0.75 \leq \eta \leq 1$. For $\eta = 1$, that is, for GR, the value of σ obtained from bulk viscosity is found to be almost the same as that obtained from the former consideration of matter and radiation energy densities at the epoch α when the particles become relativistic and contribute to the radiation energy density. Thus, the present consideration limits η to the range $0.75 \leq \eta \leq 1$.

It is interesting to note that for the lowest value of η in the above-mentioned range the present value of the matter density parameter Ω_0 (which is the ratio of the present matter energy density to the critical density) given by $\Omega_0 = \eta$ (Abdel-Rahman, 1997) is reduced to 75% of its value from a standard spatially flat RW universe with GR ($\eta = 1$). However, this lowest value can only increase the age of the universe marginally; specifically, by a factor $3/(2 + \eta) = 1.09$.

In De (1993b), the quantum production of very large massive particles (masses $\geq 53.3 m_{\mathrm{Pl}}$) was considered. These created particles at the Planck-order time give rise to the energy density at the epoch from which the expansion stage of the universe begins. From this energy density one can calculate the energy density at the time the mild inflationary phase is turned off, that is, at the transition epoch α. It is found to be $\rho(\alpha) = 2.16 \times 10^{89}$ cm^{-4}. From the ratio of the energy densities at α, as in equation (51), the matter-energy density ρ_m at α can be obtained. With the value of γ given by (54) we have $\rho_m(\alpha) = 2.16 \times 10^{71}$ cm^{-4}. From this the number density of the particles (baryons) can be found and consequently one can find the total particle number at the transition epoch with the use of the volume of the universe at that epoch obtained from the standard cosmological invariant given in (72). The total particle number is $\gtrsim 10^{78}$ and consequently the total entropy $S > 10^{87}$. In the GR formalism, these quantities remain constant after the epoch α when the standard cosmology sets in. Thus, these values of the total particle number and entropy correspond to their present values also. Now, as discussed by Blau and Guth (1987), this large value of the entropy per comoving volume might be regarded as an alternative statement of the flatness problem. In the standard cosmological models it is a matter of setting this large value of S as an initial condition, but in the present case, it achieves such a large value because of the creation phenomenon. The large value of the particle number (baryon number) of the universe is also the outcome of this model. Finally, we remark that the horizon problem should not exist in this case because the universe originates from Minkowski space-time.

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